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BLUNT LEADING EDGE OF A CYLINDRICAL WING
DURING SLIP

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ABSTRACT

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The work applies two-dimensional solution of the heat transfer problem to a three-dimensional case with gas flow under stationary conditions and constant laminar flow, Prandtl number, ^{specific}~~specific~~ heat and wing surface temperature.

The problem of heat transfer in the neighborhood of the leading critical point in a two-dimensional gas flow has been solved by A. Ye. Kalikhman (ref. 1). 22*

The present work generalizes this solution for the three-dimensional case involving the flow of gas around a cylindrical wing of infinite length during slip under stationary conditions. It is assumed that the flow in the boundary layer is laminar and that the wing surface temperature, the Prandtl number and specific heat are constant. An exact solution of the problem, when the Prandtl number is equal to one, has been obtained numerically by Ye. Reshotko (ref. 2).

*Numbers given in margin indicate pagination in original foreign text.

1. Let us consider the stationary gas flow around a wing of infinite length.

We use the following symbols:

- x, y, z - coordinates of points in the region of the boundary layer measured respectively along the arc of the airfoil from the leading edge in the plane perpendicular to the generatrices of the wing, along the normal to the airfoil and along the span parallel to the generatrices (fig. 1),
- u, v, w - velocity components along the x, y, z , axes,
- U_0, W - components along the x, z , axes of velocity \vec{U}_0 at the external boundary of the boundary layer,
- p - pressure,
- ρ - density in the boundary layer,
- ρ_0 - density at the external boundary of the boundary layer,
- T - temperature in the boundary layer,
- T_0 - temperature at the external boundary of the boundary layer,
- T_{00} - adiabatic retardation temperature,
- c_p - specific (mass) heat at constant pressure,
- μ - dynamic viscosity coefficient,
- λ - coefficient of heat transfer
- $Pr = \frac{\mu c_p}{\lambda}$ - Prandtl number
- R - gas constant
- j - mechanical equivalent of heat

In the selected system of coordinates all the flow characteristics in the boundary layer u, v, w, ρ, μ , and T depend only on x and y , while at the external boundary of the boundary layer they depend only on x^* .

*We note that $W = \text{const.}$

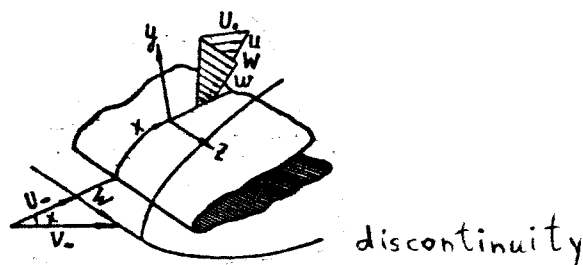


Figure 1

Consequently the system of equations for the laminar boundary layer of 23 gas in the case of stationary flow when the Prandtl number and the specific heat are constant, may be written in the following form:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = - \frac{dp}{dx} + \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right), \quad (1)$$

$$\rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} = \frac{\partial}{\partial y} \left(\mu \frac{\partial w}{\partial y} \right), \quad (2)$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0, \quad (3)$$

$$\rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial y} = \frac{1}{Pr} \frac{\partial}{\partial y} \left(\mu \frac{\partial T}{\partial y} \right) + \frac{\mu}{jc_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{u}{jc_p} \frac{dp}{dx} + \frac{\mu}{jc_p} \left(\frac{\partial w}{\partial y} \right)^2, \quad (4)$$

$$p = \rho R T, \quad (5)$$

$$\mu = f(T). \quad (6)$$

We shall solve the problem ^{using} ~~assuming~~ the following boundary conditions:

1) $u=v=w=0$, $T=T_w = \text{const}$ at the surface of the wing ($y=0$),

2) $u=U_0(x)$, $w=W=\text{const}$, $T=T_0(x)$ at the external boundary of the boundary layer ($y \rightarrow \infty$).

It is assumed that the velocity distribution at the external boundary of the boundary layer $U_0(x)$ is known in the neighborhood of the leading edge of the wing*: $U_0 = \beta x$, where β is a constant.

*This means that $p(x)$ and $T_0(x)$ are also known because according to the Bernoulli equation $\frac{dp}{dx} = -\rho_0 U_0 \frac{dU_0}{dx}$,

and according to the law of energy conservation

$$jc_p T_0 + \frac{U_0^2 + W^2}{2} = jc_p T_\infty$$

The variation in the dynamic coefficient of viscosity as a function of temperature, according to Chapmen and Rubezin (ref. 3) is given by the expression:

$$\frac{\mu}{\mu_{00}^*} = c_w \frac{T}{T_{00}^*}, \quad c_w = \left(\frac{T_w}{T_{00}^*} \right)^{\frac{1}{2}} \frac{T_{00}^* + T_s}{T_w + T_s}, \quad (7)$$

where T_{00}^* is the temperature of incomplete adiabatic retardation at the external boundary of the boundary layer (because only the velocity component U_0 decreases to zero),

μ_{00}^* is the dynamic viscosity coefficient at the temperature T_{00}^* ,

T_s is a constant which is approximately equal to 119° for air.

Since U_0 is small near the leading edge, in the future we shall:

1) neglect the terms $\frac{u}{j c_p} \frac{dp}{dx}$ and $\frac{\mu}{j c_p} \left(\frac{\partial u}{\partial y} \right)^2$ in the energy equation (4); these terms express the work performed by the pressure forces and partially the dissipation,

2) assume that $T_0 = T_{00}^*$ and $\rho_0 = \rho_{02}^*$ (ρ_{02}^* is the gas density at the external boundary of the boundary layer during incomplete adiabatic retardation due to the fact that only the velocity component U_0 decreases to zero).

The last two assumptions are analogous to those made in the work of Kalikhman (ref. 1).

2. To solve the problem we introduce a new variable $\frac{\eta}{\sqrt{x}}$ in place of y as [24] proposed by A. A. Dorodnitsyn (ref. 4):

$$\eta = \int_0^y \frac{\rho}{\rho_{02}^*} dy.$$

Then with the assumptions made above and with the selected law governing the variation in the dynamic coefficient of viscosity as a function of temperature as expressed by equation (7), the system of equations (1) - (4) will have the form:

$$u \frac{\partial u}{\partial x} + \tilde{v} \frac{\partial u}{\partial \eta} = \frac{\rho_0}{\rho} U_0 \frac{dU_0}{dx} + \tilde{v} \frac{\partial^2 u}{\partial \eta^2}, \quad (8)$$

$$u \frac{\partial w}{\partial x} + \tilde{v} \frac{\partial w}{\partial \eta} = \tilde{v} \frac{\partial^2 w}{\partial \eta^2}, \quad (9)$$

$$\frac{\partial u}{\partial x} + \frac{\partial \tilde{v}}{\partial \eta} = 0, \quad (10)$$

$$u \frac{\partial T}{\partial x} + \tilde{v} \frac{\partial T}{\partial \eta} = \frac{1}{Pr} \frac{\partial^2 T}{\partial \eta^2} + \frac{\tilde{v}}{j c_p} \left(\frac{\partial w}{\partial \eta} \right)^2, \quad (11)$$

where

$$\tilde{v} = \frac{\rho_{00}}{\rho_0} c_w, \quad \tilde{v} = u \frac{\partial \eta}{\partial x} + v \frac{\partial \eta}{\partial y}.$$

The boundary conditions for the problem under consideration will have the form:

$$1) u=v=w=0, T=T_w = \text{const when } \eta = 0,$$

$$2) u=U_0(x) = \beta x, w=W=\text{const}, T=T_0=T_{00}^* \text{ when } \eta \rightarrow \infty.$$

If we let $\frac{\rho_0}{\rho} = 1^*$ in equation (8) the system of equations (8) - (10) may be solved independently of (11). The field of velocities in the coordinates x, η , obtained from the solution of system (8)-(10) will be the same as in the well-known analogous problem for the incompressible fluid (ref. 5):

$$u = \beta \zeta f'(\zeta), \quad (12)$$

$$\tilde{v} = -\sqrt{\beta} f(\zeta), \quad (13)$$

$$w = W g(\zeta), \quad (14)$$

where $\zeta = \eta \sqrt{\frac{\beta}{\nu}}$, while the functions $f(\zeta)$ and $g(\zeta)$ are determined from the following equations and boundary conditions:

$$f'' + f''f - f'^2 + 1 = 0, \quad (15)$$

$$f(0) = f'(0) = 0, \quad f'(\infty) = 1, \quad (16)$$

$$g'' + f g' = 0, \quad (17)$$

$$g(0) = 0, \quad g(\infty) = 1. \quad (18)$$

The primes indicate derivatives with respect to ζ .

* This simplification which pertains to the determination of the field of velocities and which does not affect the field of temperature too much, was used in (ref. 1).

The result of the numerical solution of equations (15) with boundary conditions (16) may be found, for example, in (ref. 5). This reference also presents a table for functions $g(\zeta)$ and $g'(\zeta)$.

If we transform from the variable η to the variable ζ in equation (11) /25 and substitute u, v, w into it according to expressions (12), (13) and (14), we obtain:

$$f'x \frac{\partial \bar{T}}{\partial x} - f \frac{\partial \bar{T}}{\partial \zeta} = \frac{1}{Pr} \frac{\partial^2 \bar{T}}{\partial \zeta^2} + \frac{W^2}{jc_p T_{\infty}^*} g'^2, \quad (19)$$

where

$$\bar{T} = \frac{T}{T_{\infty}^*}$$

The boundary conditions will be:

- 1) $\bar{T} = \bar{T}_w = \text{const}$ when $\zeta = 0$
- 2) $\bar{T} = 1$ when $\zeta \rightarrow \infty$

Equation (19) is a linear nonhomogeneous differential equation with partial derivatives of the second order and with nonhomogeneous boundary conditions.

Its solution is obtained quite easily by separating the variables:

$$\begin{aligned} \bar{T}(\zeta) = & -Pr \bar{W}^2 \int_0^\zeta e^{-Pr \int_0^\zeta f d\zeta} \left(\int_0^\zeta g'^2 e^{Pr \int_0^\zeta f d\zeta} d\zeta \right) d\zeta + \\ & + (1 - \bar{T}_w) \frac{\int_0^\zeta e^{-Pr \int_0^\zeta f d\zeta} d\zeta}{\int_0^\infty e^{-Pr \int_0^\zeta f d\zeta} d\zeta} + \\ & + Pr \bar{W}^2 \int_0^\infty e^{-Pr \int_0^\zeta f d\zeta} \left(\int_0^\zeta g'^2 e^{Pr \int_0^\zeta f d\zeta} d\zeta \right) d\zeta \frac{\int_0^\zeta e^{-Pr \int_0^\zeta f d\zeta} d\zeta}{\int_0^\infty e^{-Pr \int_0^\zeta f d\zeta} d\zeta} + \bar{T}_w, \end{aligned} \quad (20)$$

where

$$\bar{W}^2 = \frac{W^2}{jc_p T_{\infty}^*}$$

3. The heat flux from the gas to the wing surface is equal to:

$$q_w = \lambda_w \left(\frac{\partial T}{\partial \eta} \right)_{\eta=0} = \lambda_w \frac{p_w}{p_{02}} T_{00} \sqrt{\frac{\beta p_{02}}{\mu_{00} c_w}} \left(\frac{\partial \bar{T}}{\partial \zeta} \right)_{\zeta=0} \quad (21)$$

Differentiating equality (20) with respect to ζ and letting $\zeta = 0$, we obtain:

$$\begin{aligned} \left(\frac{\partial \bar{T}}{\partial \zeta} \right)_{\zeta=0} = & (1 - \bar{T}_w) \frac{1}{\int_0^\infty e^{-Pr \int_0^\zeta f d\zeta} d\zeta} + \\ & + Pr \bar{W}^2 \int_0^\infty e^{-Pr \int_0^\zeta f d\zeta} \left(\int_0^\zeta g^2 e^{Pr \int_0^\zeta f d\zeta} a \zeta \right) d\zeta \frac{1}{\int_0^\infty e^{-Pr \int_0^\zeta f d\zeta} d\zeta}. \end{aligned} \quad (22)$$

Expression (22) for Prandtl numbers from 0.6 to 1 may be approximated /26
with a maximum error of 1 percent in the following manner:

$$\left(\frac{\partial \bar{T}}{\partial \zeta} \right)_{\zeta=0} \cong 0.570 Pr^{0.4} (1 - \bar{T}_w) + 0.285 Pr^{0.9} \bar{W}^2. \quad (23)$$

Then

$$q_w \cong \lambda_w \frac{p_w}{p_{02}} T_{00} \sqrt{\frac{\beta p_{02}}{\mu_{00} c_w}} [0.570 Pr^{0.4} (1 - \bar{T}_w) + 0.285 Pr^{0.9} \bar{W}^2]. \quad (24)$$

Since

$$\begin{aligned} \bar{W}^2 &= 2 \left(\frac{T_{00}}{T_w} - 1 \right), \\ \frac{p_w}{p_{02}} &= \frac{T_{00}}{T_w}, \quad \mu_{00} c_w = \mu_w \frac{T_{00}}{T_w}, \end{aligned}$$

we have

$$q_w \cong 0.570 Pr^{0.4} \lambda_w \sqrt{\frac{\beta p_w}{\mu_w}} [(T_{00} - T_w) + Pr^{0.5} (T_{00} - T_{00}^*)]$$

or

$$q_w \cong 0.570 Pr^{0.4} \lambda_w \sqrt{\frac{\beta p_w}{\mu_w}} T_{00} \left(\frac{1 + Pr^{0.5}}{1 + w} - \frac{T_w}{T_{00}} \right), \quad (25)$$

where

$$\alpha = \frac{\frac{\gamma-1}{2} M_\infty^2 \sin^2 \chi}{1 + \frac{\gamma-1}{2} M_\infty^2 \cos^2 \chi}$$

M_∞ is the M number of the unperturbed flow

χ is the angle of slip of the wing, $\alpha = \frac{c_p}{c_\tau}$.

4. Let us compare the solution which we have obtained with the exact one proposed by Reshotko (ref. 2) for the case $Pr = 1$.

When $Pr = 1$ expression (25) will have the form:

$$q_w \cong 0.570 \lambda_w \sqrt{\frac{\beta \rho_w}{\mu_w}} T_\infty \left(1 - \frac{T_w}{T_\infty}\right) \quad (26)$$

According to the exact equation

$$q_w = K \lambda_w \sqrt{\frac{\beta \rho_w}{\mu_w}} T_\infty \left(1 - \frac{T_w}{T_\infty}\right),$$

where

$$K = f\left(\frac{T_w}{T_\infty}, \alpha\right).$$

The values of K were obtained by Reshotko (ref. 2) by a combined /27
numerical solution of two ordinary differential equations with $\frac{T_w}{T_\infty} = 2-1-0$ and
with different ω .

For the case of the greatest practical interest $0 < \frac{T_w}{T_\infty} < 1$ the error in the
calculation of the heat flux from the gas to the surface of the body by means of
the approximate equation (26) does not exceed 16 percent if $\omega < 2$, i.e., for
example, when $M_\infty = 5$ $\chi < 65^\circ$ or when $M_\infty = 4$ $\chi < 70^\circ$.

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